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GENERAL SUSTAINABLE HARVESTING MODELS WITH ENVIRONMENTAL STOCHASTICITY

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In a randomly varying environment, we model the growth of the harvested population by a very general stochastic differential equation (SDE) model, where an also very general sustainable harvesting policy is applied: $dX(t) = g(X(t))X(t)dt - qE(X(t))X(t)dt + \sigma(X(t))X(t)dW(t)$ with $X(0) = x_0 > 0$, where $W(t)$ is a standard Wiener process.

Use of very general models has the advantage of obtaining results and desirable properties of sustainable harvesting policies that are model robust. Results on this general model have already been presented in [1, 2]; they are here reviewed and extended, as well as applied to optimization issues.

Stratonovich calculus is used for a more convenient interpretation, but attention is called to the fact that results are equivalent if one uses Itô calculus (see [2]). We consider the natural growth of the harvested population to be of a very general density-dependent form, with a geometric average *per capita* natural growth rate $g(x)$ being a C^1 strictly decreasing function and with $\sigma(x)\varepsilon(t)$ (where $\sigma(x)$ is a positive C^2 noise intensity and $\varepsilon(t) = dW(t)/dt$ is a standard white noise) describing the effect of environmental random fluctuations on that rate. The catchability coefficient is $q > 0$ and $E(x)$ is a non-negative C^1 function representing a very general sustainable harvesting effort when population size is x.

We show existence and uniqueness of the solution and determine the conditions for non-extinction of the population and existence of a stationary density, i.e. for the existence of a sustainable stochastic equilibrium. Results are then applied to optimization issues for particular harvesting policies.

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