

SUSTAINABLE FISHERIES MANAGEMENT IN RANDOM ENVIRONMENTS: FOX MODEL

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To describe the growth of a harvested population when the environment is subjected to random fluctuations one can use Stochastic Differential Equation models as in [1] and [2]. Here we consider a Gompertz model for the average natural growth to which we subtract a harvesting yield term based on a constant or variable fishing effort.

There is previous work on the optimal design of the harvesting policy with the purpose of maximizing the expected accumulated profit (discounted by a depreciation rate) over a finite time horizon (see, for instance, [3]).

We consider a quite general profit structure which includes linear prices per unit yield and linear costs per unit effort. The harvesting efforts of the optimal policies vary with the randomly varying population size and such policies can, under certain conditions, even be of bang-bang type. These policies are not applicable to harvesting since they need population size to be constantly evaluated and require very frequent randomly determined changes in harvesting effort.

Our approach, based on sustainable and applicable fishing policies, leads to sustainability of the population and to a stationary distribution of the population size and does not require evaluation of population size (some previous work regarding these approach can be seen in [4] and in [5]). We consider constant harvesting effort policies (Fox model) and determine the effort that optimizes the expected sustainable profit per unit time. We check what we lose profitwise by using this policy instead of the optimal inapplicable policy with variable effort. Using Monte Carlo simulations, we show that, for common situations, our approach is almost as profitable as the first.

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