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NON-LINEAR EVOLUTIONARY MATRIX MODELS WITH MULTIPLE TRAIT

F. Martins^{1*}, J. M. Cushing^{2,3}, A. A. Pinto¹ and A. Veprauskas³

¹Department of Mathematics, Faculty of Sciences
University of Porto and LIAAD-INESC

²Department of Mathematics, University of Arizona

³Interdisciplinary program in Applied Mathematics
University of Arizona

philip_m90@hotmail.com (*corresponding author), cushing@math.arizona.edu
aapinto1@gmail.com, aveprauskas@math.arizona.edu

One fundamental question in biology is population extinction and persistence, i.e., stability/instability of the extinction equilibrium and of non-extinction equilibria. In the case of non-linear matrix models for structured populations, a bifurcation theorem answers this question when the projection matrix is primitive by showing the existence of a continuum of positive equilibria that bifurcates from the extinction equilibrium as the inherent population growth rate passes through 1. This theorem also characterizes the stability properties of the bifurcating equilibria by relating them to the direction of bifurcation, which is forward (backward) if, near the bifurcation point, the positive equilibria exist for inherent growth rates greater (less) than 1. In this paper we consider an evolutionary game theoretic version of a general nonlinear matrix model that includes the dynamics of a vector of mean phenotypic traits subject to natural selection. We extend the fundamental bifurcation theorem to this evolutionary model. We apply the results to an evolutionary version of a Ricker model with an added Allee component. This application illustrates the theoretical results and, in addition, several other interesting dynamic phenomena, such as backward bifurcation induced strong Allee effects.

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